

Brazil-Mexico 2<sup>nd</sup> meeting on Singularities  
and  
3<sup>rd</sup> Singularity Theory Meeting of Northeast region  
From 08 to 17 July, 2015



Institute of Mathematics - Federal University of Bahia  
Salvador - Bahia - Brazil

The *2nd* Brazil-Mexico Meeting on Singularities is an international meeting scheduled to be held every two years, in odd years, alternating between the countries Brazil and Mexico. The aim is to create a new common space for researchers in Singularity theory and Foliation theory, from both countries, to discuss and present their recent progress in these areas.

The Brazilian Northeastern Meeting on Singularities is also an international meeting, scheduled to be held every two years, in odd years. The purpose of the meeting is to bring together in the Brazilian Northeastern region some of the leading researchers from Brazil and from abroad, to promote the discussion of the latest issues concerning Singularity Theory and affine areas, such as Algebraic Geometry, Bifurcation Theory and Dynamical Systems.

Both meetings leads to an important interaction between experienced researchers, young researchers and graduate students from various regions of Brazil and abroad.

# Brazil-Mexico 2nd Meeting on Singularities - 2015

## Title and Abstract of Talks

### Monday - July 13

- Speaker: Jawad Snoussi, UNAM - Mexico

Title: Milnor fibrations and regularity conditions for real analytic mappings.

Abstract: When  $f : (\mathbb{R}^n, 0) \rightarrow (\mathbb{R}^p, 0)$  is a surjective real analytic map with isolated critical value, We describe how some regularity conditions such as: Thom's  $a_f$ ,  $c$ -regularity,  $d$ -regularity and  $m$ -regularity, play a role on ensuring Milnor fibration conditions either on the tube or the sphere.

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- Speaker: Alice Kimie Miwa Libardi, UNESP - Brazil

Title: Functions on  $C(\mathbb{C}P^n)$ -singular manifolds

Abstract: This work is a joint work with V.V. Sharko. A manifold with isolated singularities is a topological space  $M$  which has the structure of a smooth ( $C^\infty$ ) manifold in  $M \setminus S$ , where  $S$  is the discrete set of singular points of  $M$ . A diffeomorphism between two such manifolds  $M$  and  $N$  is a homeomorphism from  $M$  into  $N$  such that sends the set of singular points of  $M$  onto the set of singular points of  $N$  and is a diffeomorphism outside of them. We say that  $M$  has a cone-like singularity at a (singular) point  $P \in S$  if there exists a neighbourhood of the point  $P$  diffeomorphic to a cone over a smooth manifold  $W_P$  ( $W_P$  is called the link at the point  $P$ ). In what follows we assume all manifolds have only cone-like singularities. In general, a  $C(\mathbb{C}P^n)$ -singular manifold is obtained from a smooth  $(2n+1)$ -manifold with boundary which is a disjoint union of complex projective spaces  $\mathbb{C}P^n \cup \mathbb{C}P^n \cup \dots \cup \mathbb{C}P^n$  and subsequent capture of the cone over each component of the boundary. For this type of  $C(\mathbb{C}P^n)$ -singular manifold parity of the number of singular points depends on parity of the number  $n$ . We give

some results on manifolds with semi-free  $S^1$ -action which has only isolated fixed points and study functions on  $C(\mathbb{C}P^n)$ -singular manifolds. We prove the following result: Theorem : Let  $M^{2n+1}$ , ( $2n \geq 5$ ), be a compact simply connected  $C(\mathbb{C}P^n)$ -singular manifold with singular points  $m_1, \dots, m_k$ . Let  $\sigma$  be a permutation of  $(1, 2, \dots, k)$  and let  $A$  (with  $s$  points) and  $B$  (with  $k - s$  points) be the split of the singular points  $m_1, \dots, m_k$  into two disjoint sets :  $A = m_{\sigma(1)}, m_{\sigma(2)}, \dots, m_{\sigma(s)}$ ,  $B = m_{\sigma(s+1)}, m_{\sigma(s+2)}, \dots, m_{\sigma(k)}$ . We fix a collection of almost Morse functions  $St = \underbrace{\pi_*(f_1), \pi_*(f_1), \dots, \pi_*(f_1)}_s, \underbrace{\pi_*(f_2), \pi_*(f_2), \dots, \pi_*(f_2)}_{k-s}$  in the neighborhoods  $U(m_{\sigma(1)}), U(m_{\sigma(2)}), \dots, U(m_{\sigma(s)}), U(m_{\sigma(s+1)}), \dots, U(m_{\sigma(k)})$  respectively, where  $f_1 = \sum_{i=1}^{2n} |z_i|^2$ ,  $f_2 = 1 - \sum_{i=1}^{2n} |z_i|^2$ . Then  $\mathcal{M}_\lambda(M^{2n+1}, St) = \mu(H_\lambda(M^{2n+1} \setminus B, A, \mathbb{Z})) + \mu(TorsH_{\lambda-1}(M^{2n+1} \setminus B, A, \mathbb{Z}))$ , where  $\mu(H)$  is the minimal number of generators of the group  $H$ .

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- Speaker: Mirna Lisette Gomez Morales, Centro de Innovación Matemática - Mexico.

Title: The codimension-one discriminant of a stable mapping.

Abstract: We study the  $\mathcal{A}_e$ -codimension one map-germs that may be obtained by restricting the target of a minimal stable map-germ  $F : (\mathbb{C}^n, 0) \rightarrow (\mathbb{C}^p, 0)$ . We call the set of hyperplanes  $\{L : \text{codim}_{\mathcal{A}_e}(F|_L) > 1\}$  the codimension-one discriminant of the stable map  $F$  and prove that it defines an analytic variety  $V(I_F)$  in the space of hyperplanes, visualised inside  $\mathbb{C}^p$ .

Moreover, we describe the codimension-one discriminant of a number of minimal stable mappings, focusing on unfoldings of map-germs from  $\mathbb{C}^2$  to  $\mathbb{C}^3$ . Although, in principle, the codimension-one discriminant seems to be a linear free divisor, we show that this property does not hold for every minimal stable map-germ  $F$ .

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- Speaker: Lê Dung Trang

Title: Linear bundles with connections.

Abstract: The group of isomorphism class of linear bundles on an algebraic variety is known as the Picard group of the algebraic variety.

We consider isomorphism classes of linear bundle with connections in the case of non-singular varieties. This is a joint work with H. Hamm.

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- Speaker: Osamu Saeki, Kyushu University - Japan.

Title: Cobordism group of Morse functions on surfaces with boundary.

Abstract: We consider Morse functions on compact manifolds possibly with boundary, and define their admissible cobordism group, based on generic maps into the plane that are submersions near the boundary. Then, we show that the cobordism group of Morse functions on surfaces with boundary is isomorphic to the cyclic group of order two. This is a joint work with Takahiro Yamamoto (Kyushu Sangyo Univ., Japan).

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- Speaker: Andrés Daniel Duarte, Universidad Autónoma de Zacatecas - Mexico.

Title: A higher-order version of Nobile's theorem.

Abstract: The higher Nash blowup is a natural modification of an algebraic variety that replaces singular points by limits of certain vector spaces associated to non-singular points. A. Nobile proved that the usual Nash blowup of a variety is an isomorphism if and only if the variety is non-singular. In this talk we will discuss a higher-order version of this theorem.

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- Speaker: Rodrigo Mendes Pereira, Federal University of Ceará - Brazil

Title: Knots and the topology of singular surfaces in  $\mathbb{R}^4$ .

Abstract: We consider the link of an isolated surface singularity in  $\mathbb{R}^4$  given as the image of an analytic map germ  $f : (\mathbb{R}^2, 0) \rightarrow (\mathbb{R}^4, 0)$ . The link is a knot in a small enough sphere which determines the topological type of the singularity. We prove that the link of a generic linear projection into  $\mathbb{R}^3$  is equivalent to a generic projection of the link of  $f$ . We also give sufficient conditions for the knot type is trivial. In addition, we show that nontrivial knots do not appear, in this case, as link of normally embedded surfaces. We will also discuss generalizations of this result.

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- Speakers: Carles Bivià Ausina, Universitat Politècnica de València - Spain.

Title: Log-canonical threshold of ideals and the sequence of mixed multiplicities

Abstract: We characterize the ideals  $I$  of finite colength of the ring  $\mathcal{O}_n$  of complex analytic functions germs  $(\mathbb{C}^n, 0) \rightarrow \mathbb{C}$  whose integral closure is equal to the integral closure of an ideal generated by pure monomials. This characterization, which is motivated by an inequality proven by Demailly and Pham, is given in terms of the log canonical threshold of  $I$  and the sequence of mixed multiplicities of  $I$ . Moreover, we relate this topic with the question of characterizing the ideals of  $\mathcal{O}_n$  whose multiplicity is equal to the product of the sequence of mixed Łojasiewicz exponents of  $I$ .

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- Abramo Hefez, Federal Fluminense University - Brazil.

Title: The Milnor Number in Positive Characteristic.

Abstract: In this talk we will discuss some properties of the Milnor number for hypersurfaces defined over fields of positive characteristic. At the end we focus on plane branches where we compare Milnor's number with the delta invariant. Joint work with J.H.O. Rodrigues and R. Salomão.

## Tuesday - July 14

- Speaker: Federico Sanchez Bringas, UNAM - Mexico.

Title: Geometry of  $G$ -Deformations of Immersions of the Plane into  $\mathbb{R}^4$

Abstract: Let  $f, \bar{f}$  be non-totally umbilic immersions of a surface  $M$  into  $\mathbb{R}^4$  with Gauss maps that differ by a reverse-orientation congruence in the Grassmannian. Such an  $\bar{f}$  is called a  $G$ -deformation of  $f$ . This type of deformations are uniquely determined, up to the composition of an homothety and a translation. We describe the second order invariants and asymptotic lines of such immersions and their  $G$ -deformations when  $M = \mathbb{R}^2$ . Interesting examples appear in this setting. Joint work with J. Monterde.

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- Speaker: Mostafa Salarinoghabi, ICMC/USP/São Carlos - Brazil.

Title: Flat and Round Singularity theory of plane curves.

Abstract: We study singular plane curve and the way they bifurcate in families of curves. The bifurcations include the information on the inflections and vertices of the curve. We also obtain information about the evolute of the curve and of its bifurcations. We deal with the cases when the curve has a cusp or a ramphoid cusp. For this we need information about the evolute at first and second order inflections.

- Speaker: Faustino Agustín Romano Velázquez, UNAM - Mexico.

Title: A isotopy theorem for isolated polar weighted homogeneous singularities.

Abstract: In this talk we present a isotopy theorem for isolated polar weighted homogeneous singularities, this theorem will be used in the classification of isolated polar weighted homogeneous singularities. This is join work with José Luis Cisneros-Molina.

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- Speaker: Marcos Craizer, PUC - Rio - Brazil

Title: Equiaffine Characterization of Lagrangian Surfaces in  $\mathbb{R}^4$ .

Abstract: For non-degenerate surfaces in  $\mathbb{R}^4$ , there is a distinguished transversal bundle called affine normal plane bundle. Lagrangian surfaces have remarkable properties with respect to this normal bundle, like for example, the normal bundle being Lagrangian. In this talk we discuss a characterization of surfaces which are Lagrangian with respect to some parallel symplectic form in  $\mathbb{R}^4$ .

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- Speaker: Jean-Paul Brasselet, Aix-Marseille Université - France.

Title: How curious it is and what a coincidence!

Subtitle: **Mrs M:** *That is curious! How very bizarre! And what a coincidence! I took the same train, sir I too.*

**Mr M:** *How curious that is and what a coincidence! We were facing each other, my dear lady!*

The bald soprano.

**Eugène Ionesco. 1947**

Abstract: The famous Lefschetz fixed points theorem provides a way to measure in what extent a map  $f : M \rightarrow M$  from a compact manifold to itself has fixed points, *i.e.* points such that  $f(x) = x$ . The theorem has a lot of applications in and out of mathematics;

On the one hand the theorem has been extended to so-called “coincidence points” of two functions  $f$  and  $g$  from a compact manifold to itself. Coincidence points are points such that  $f(x) = g(x)$ . Lefschetz theorem is the particular case  $g = \text{id}_M$ .

On the other hand Goresky and MacPherson extended the Lefschetz theorem in the case of singular spaces, using intersection homology theory that they created.

In this talk, I will present recent results obtained on the one side with Tatsuo Suwa, from Hokkaido University and on the other side with Marcelo Saia (USP, São Carlos), Alice K. Libardi, Thais F. Monis and Eliris C. Rizzioli (UNESP, Rio Claro), providing generalisation of the previous results for coincidence of maps between singular spaces.

**Mr Martin:** *How curious it is, how curious it is, how curious it is, and what a coincidence! You know, in my bedroom there is a bed, and it is covered with a green eiderdown. This room, with the bed and the*

*green eiderdown, is at the end of the corridor between the w.c. and the bookcase, dear lady!*

**Mrs Martin** *What a coincidence, good Lord, what a coincidence! My bedroom, too, has a bed with a green eiderdown and it is at the end of the corridor, between the w.c., dear sir, and the bookcase!*

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- Speaker: Fuensanta Aroca Bisquert, UNAM - Mexico.

Title: TBA.

Abstract: TBA.

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- Speaker: Kazuto Takao, Institute of Mathematics for Industry, Kyushu University - Japan.

Title: Heegaard splittings and singularities of product maps.

Abstract: Singularities of the product map of two smooth functions on a manifold can be used to analyze the relationship between the functions. This idea originated in 3-dimensional topology, and has been successfully applied to study Heegaard splittings of 3-manifolds. In this talk, I will survey relevant results, singularity theoretic problems posed by them, and my approach to the problems.

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- Speaker: Christophe Eyrat, Institute of Mathematics, Polish Academy of Sciences - Poland.

Title: Topologically equisingular deformations of homogeneous hypersurfaces with line singularities are equimultiple.

Abstract: I will show that if  $\{f_t\}$  is a family of line singularities with constant Lê numbers and such that  $f_0$  is a homogeneous polynomial, then  $\{f_t\}$  is equimultiple. This extends to line singularities a well known theorem of A. M. Gabrièlov and A. G. Kušnirenko concerning isolated singularities. As an application, I will show that if  $\{f_t\}$  is a topologically  $V$ -equisingular family of line singularities, with  $f_0$  homogeneous, then  $\{f_t\}$  is equimultiple. This provides a new partial positive answer to the famous Zariski multiplicity conjecture for a special class of non-isolated hypersurface singularities.

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- Speaker: María del Carmen Romero Fuster, Facultat de Matemàtiques, Universitat de València - Spain.

Title: Topology and singularities in the second order geometry of 3-manifolds.

Abstract: We analyze the second order geometry properties of a 3-manifold  $M$  immersed in  $\mathbb{R}^6$  by studying the different possible models of its curvature locus at each point. This object, which is completely determined at each point by the second fundamental form (and conversely, it completely determines the second fundamental form) of  $M$  is a linear projection of a Veronese surface in the normal space of the 3-manifold. We call it the *curvature Veronese* at the considered point. We describe all the possible shapes for the curvature Veronese on 3-manifolds immersed in  $\mathbb{R}^6$ . We analyze the connections between the properties of the curvature Veronese, the principal configurations and certain singularities of height and squared distance functions on the manifold.

An interesting fact is that the curvature Veronese is an algebraic surface with singularities which is contained in the (3-dimensional) normal space of  $M$  at the point and we can interpret these singularities in terms of the behaviour of the family of principal configurations at each point. Moreover, we describe how to detect the local convexity of the 3-manifold in terms of the relative position of the curvature Veronese with respect to origin of the normal space.

## Wednesday - July 15

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- Speaker: Aurélio Menegon Neto, Federal University of Paraiba - Brazil.

Title: Lê's Polyhedron for Line Singularities.

Abstract: We study the topology of line singularities, describing the degeneration of its Milnor fiber to the singular hypersurface by means of a vanishing polyhedron and a collapsing map.

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- Speaker: Jonny Ardila Ardila, Federal University of Rio Janeiro - Brazil.

Title: On Complex Vector Fields With Isolated Singularities.

Abstract: We will show results obtained in integrability of singular holomorphic vector fields in  $\mathbb{C}^3$  and the machinery necessary for attain them, that goes from simple explorations, to new results in: formal series, formal diffeomorphisms, groups of diffeomorphisms, blow-ups and formal first integrals.

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- Speaker: Edwin León Cardenal, CIMAT - Mexico.

Title: Local Zeta Functions for Degenerate Polynomials.

Abstract:  $p$ -adic local zeta functions are related with the number of solutions of congruences modulo  $p^m$  and with exponential sums modulo  $p^m$ . There are several conjectures and intriguing connections between local zeta functions and singularity theory. Classically one may study the local zeta function attached to a polynomial (or analytic function) by using a Newton polygon and some non-degeneracy condition. In this talk we study a new non-degeneracy condition introduced by Saia and Zúñiga-Galindo for the curve case. We also discuss some possible generalizations of their results.

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- Speaker: Nguyen Thi Bich Thuy, UNESP - Brazil.

Title: On a singular set associated to a polynomial mapping  $G : \mathbb{C}^n \rightarrow \mathbb{C}^{n-1}$ .

Abstract: We construct a real variety  $V_G$  associated to a polynomial mapping  $G : \mathbb{C}^n \rightarrow \mathbb{C}^{n-1}$  ( $n \geq 2$ ) such that if the bifurcation set  $B(G)$  of  $G$  is not empty, then the intersection homology of  $V_G$  is not trivial. This is a joint work with Maria Aparecida Soares Ruas.

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- Speaker: Ernesto Rosales Gonzalez, UNAM - Mexico.

Title: Local analytic classification of singular foliations in the complex plane.

Abstract: In this talk we consider the formal and analytical classification of generic germs of holomorphic foliations in a neighborhood of a singular point in the complex plane. Formal and analytic invariants are given.

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- Speaker: Alexandre César Fernandes Gurgel, Federal University of Ceará - Brazil.

Title: TBA.

Abstract: TBA.

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- Speaker: Luis Núñez-Betancourt, University of Virginia - USA.

Title: F-pure and log-canonical thresholds of graded rings.

Abstract: Both the F-pure and log-canonical threshold are invariants that measure the singularity of a variety embedded in a space. Although they arise in different characteristics and they have different natures, both invariants are strongly related. In this talk we will relate these numbers to the a-invariant, which is a classical invariant for graded rings. This result solves a conjecture posted by Hirose, Watanabe, and Yoshida. In addition, we will discuss an interpretation of the a-invariant for Gorenstein graded K-algebras in terms of regular sequences that preserve F-purity. This result is in the spirit of Bertini theorems for projective varieties. This is joint work with Alessandro De Stefani.

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- Speaker: Maria Elenice Rodrigues Hernandez, State University of Maringá - Brazil.

Title: Polar Multiplicities of Ruled Hypersurface.

Abstract: Our purpose in this work is to compute the polar multiplicities of a  $(n - 1)$ -ruled hypersurface  $X \subset \mathbb{C}^{n+1}$ , that is defined as follows. Let  $D \subset \mathbb{C}$  be a disc centered at the origin. An  $(n - 1)$ -ruled hypersurface in  $\mathbb{C}^{n+1}$  of complex dimension  $n$  is (locally) the image of a smooth map-germ  $f : (\mathbb{C}^{n-1} \times D, 0) \rightarrow (\mathbb{C}^{n+1}, 0)$  given by

$$f(x_1, \dots, x_{n-1}, t) = \alpha_0(t) + x_1\alpha_1(t) + \dots + x_{n-1}\alpha_{n-1}(t)$$

such that the map  $\alpha_i : D \rightarrow \mathbb{C}^{n+1}$  is smooth for each  $i = 0, \dots, n - 1$ . If each  $\alpha_i$  is non constant we called  $\alpha_0$  the base curve and  $\alpha_i$ ,  $i = 1, \dots, n-1$  the directrices curves. We prove that the polar multiplicities at the origin of  $X$  can be calculated in terms of the multiplicities of the generating curves of  $X$ . As a consequence we obtain the Euler obstruction of  $X$ . This is a joint work with M.E. Hernandez and R. Martins.

- Speaker: Marcio Soares, Federal University of Minas Gerais - Brazil.

Title: Local Brunella's Alternative.

Abstract: M. Brunella formulated a question, still unsolved, about the structure of codimension one holomorphic foliations on  $\mathbb{P}_{\mathbb{C}}^3$ . We will describe what is known in relation to this question and present a recent result, of local nature, which allows us to give a precise statement of a local version of Brunella's alternative: if we do not have an invariant surface, all the leaves contain a germ of analytic curve or it is possible to detect the so called nodal components in the generic points of the singular curves before doing the reduction of singularities.

- Speaker: Marcelo Escudeiro Hernandez, State University of Maringá - Brazil

Title: On the tangent space of  $\mathcal{A}$ -orbit of a quasi-ordinary hypersurface.

Abstract: In this talk we present some characterizations of the tangent vectors to the orbit of an irreducible quasi-ordinary hypersurface in

$\mathbb{C}^{r+1}$  with respect to the  $\mathcal{A}$ -equivalence. In particular, we related the tangent vectors with  $r$ -form differential of Kähler. This allow us to define a set  $\Lambda \subset \mathbb{N}^r$  that generalizes an important analytic invariant of plane branches.

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- Speaker: Mutsuo Oka, Tokyo University of Science - Japan.

Title: On the number of roots of a mixed polynomial.

Abstract: We give a generalization of Rhie on the number of roots of Lens equation. We consider a generalized Lens equation  $\bar{z} = \frac{p(z)}{q(z)}$  with  $\deg p(z) \leq n - 1, \deg q(z) = n$ . We count the number of roots without sign.

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- Speaker: Leonardo Meireles Câmara, Federal University of Espirito Santo - Brazil.

Title: Transversely holomorphic structures with many invariant hypersurfaces.

Abstract: In this talk we review some results about classical Darboux integrability and show that a transversely holomorphic foliation on a compact complex manifold  $M$  having an infinite number of invariant hypersurfaces admits a meromorphic first integral  $f : M \rightarrow \overline{\mathbb{C}}$ . *Joint work with B. Scárdua.*

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- Speaker: Alexey Remizov, ICMC/USP/São Carlos - Brazil.

Title: Singularities of geodesic flows in pseudo-Riemannian metrics.

Abstract: The talk is devoted to geodesics in two-dimensional pseudo-Riemannian metrics, i.e., metrics that change signature due to vanishing discriminant. The equation of the geodesic flow has singularities at points where the metric degenerates.

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- Speaker: José Antonio Seade Kuri, UNAM - Mexico.

Title: On the topology of singularities.

Abstract: The classical ideas of Lefschetz, Thom and others teach us about the importance of studying the topology of analytic sets by taking slices with general hyperplane sections. A natural idea is taking slices by the level hypersurfaces of a function with an isolated singularity. This has been done in various settings by several authors. In this talk we speak about some old and new results in this direction.

## Friday - July 17

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- Speaker: José Luis Cisneros Molina, UNAM - Mexico.  
Title: Classification of isolated polar weighted homogeneous singularities.  
Abstract: In this talk we present a classification of isolated polar weighted homogeneous singularities, it is a generalization of the families of complex weighted homogeneous polynomials given by Orlik and Weigreich. This is joint work with Agustín Romano-Velázquez

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- Speaker: Jessica Jaurez, UNAM - Mexico.  
Title: TBA.  
Abstract: TBA.

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- Speaker: Roberta Godoi Wik Atique, ICMC/USP/São Carlos - Brazil.  
Title: Lifiable vector fields over corank 1 multigerms.  
Abstract: I shall present a systematic method to construct liftable vector fields over an analytic multigerm of corank at most one admitting a one parameter stable unfolding.

## Poster Presentations

- **Wednesday, July 08:** Breno, Camila, Carlos Rodrigo, Daiane Henrique, Erica Boizan, Isabelly Camila.
- **Thursday July 09:** Otoniel da Silva, Oziel Martinez, Mauro Hernandez, Francisco Inuma, Andrea Sacramento.
- **Friday July 10:** Edison M .de Guzman, João Costa, Jesus Márquez, Luis Sanchez.
- **Monday July 13:** Aldicio, Miriam Pereira, Miriam Ramirez, Patricia Tempesta, Nivaldo Grulha, Liliana Olga.
- **Tuesday 14:** Ady Cambraia, Ana Claudia, Gonzalo Barranco, Liliana Costa, Masayuki Kawashima, Francisco Miguel.
- **Wednesday 15:** Sergey, Thais Fernanda, Taciana Oliveira, Mirna Lissete, Northon Canevari, Kazumasa Inaba.